

Phasor diagrams

Dear Sir—I am grateful to Mr. C. B. Gray for again showing sufficient interest in this subject to respond to a communication of mine on it.

My black-box problem was intended to be almost a rhetorical question, inasmuch as no prolonged consideration was expected to be needed to arrive at the two possibilities whose derivation Mr Gray sets forth in full. Its object was to embarrass those whose philosophy relies on being able to classify the elements of a circuit into sources and loads, since according to their principles they would be unable even to begin. By any external tests, either box could contain the source and the other the load. (If we were not told that one box contained a source, there would of course be a third possibility: an inductor in one box and a capacitor in the other.)

This response to my problem has, as I had hoped, evoked a statement of just those features in much current electrical engineering teaching that seem to be ill conceived.

Mr Gray's Fig. F shows between terminals *c* and *d* an inductor flanked by two oppositely pointing arrows, one marked *E* and the other *V*, which appear also in the corresponding phasor diagram. Neither he nor anybody else has explained to my satisfaction why it should be considered necessary to identify these two quantities of opposite sign when in fact there is only one voltage between *c* and *d* (or any other two points). During nearly 50 years of practical and theoretical study of electrical circuits of many kinds I have never even begun to feel the need for this duplication, but on the contrary find it mightily confusing.

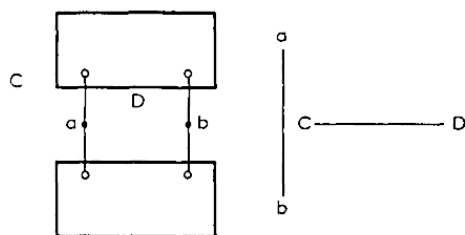
All one needs is a single quantity, difference of potential, which can be either positive or negative between any two points, say *a* and *b*. The change in potential on moving from *a* to *b* can conveniently be denoted by $v_{ab} (= v_b - v_a = -v_{ba})$. Then if current is flowing from *a* to *b*, positive v_{ab} (due to what is known as an e.m.f.) means electrical energy coming into existence in the part of the circuit between *a* and *b*, so that the circuit is thereby gaining electrical energy. Negative v_{ab} means negative power; i.e. electrical energy coming into that part of the circuit is being converted into some other form, temporarily or permanently. One does not have to start by classifying one element as a generator and another as a load and varying the treatment accordingly. Whether any element can rightly be called a source or a load is determined entirely by the sign of v in the direction of current flow, i.e. by the sign of the power $v_{ab}i_{CD}$ (or $ab.CD$ as it can more conveniently be written), *CD* being any particular current flowing from *a* to *b*.*

This principle, taught first for d.c., can be applied later without modification to a.c. (instantaneous values). For convenience, r.m.s. values, *V* and *I*, are commonly used, and techniques have been established for retaining the same forms for them as for d.c. The essential power principle still holds: if the component of *V* in phase with the current

in the same direction through a circuit element is positive, that element is a source, no matter what it may be called on the label. A quadrature component signifies power alternating between that element and the rest of the circuit. All circuit elements can be submitted to the same criterion; there is no need to treat them differently or to introduce *E* as well as *V* in the same element.

In Mr. Gray's Fig. E we see two opposite currents in one wire, represented by two oppositely pointing phasors. (Fig. C shows only one current in the same wire.) It is true that an alternating current is made up of equal numbers of halfcycles flowing in opposite directions. But there is no need to use two phasors to represent them. Doing so is a result of the curious custom of drawing an arrowhead at one end of each phasor; presumably somebody started it in the erroneous belief that the thing was a vector and everyone has followed blindly. It necessitates making an arbitrary choice of one of two identical lots of halfcycles, and leads to questions about which circuit element is source and which is load, and to anomalies such as arrows pointing in the same direction in the circuit diagram to represent quantities that are in quadrature.

Mr Gray's Figs. G and H, referring to the same situations as C and E, show only one current for both, but for the inductor case the *V* and *E* phasors, previously opposite, are now in phase, while in the capacitor case they remain opposite. Imagine a student, coming to all this for the first time, trying not to be confused. And the duality between inductance and capacitance is spoilt.



A The black-box problem with its associated phasor diagram

The conventions I proposed are free from these anomalies, embarrassments, confusions and contradictions. Fig. A shows the problem as stated, and corresponds to Mr. Gray's Fig. A. Filling in the contents of the boxes for either solution makes absolutely no other difference to the diagrams. Why should it? The electrical situation so far as external data are concerned is unchanged; why then should it be considered necessary to draw four more pairs of diagrams with the signs and symbols all different? No further useful information is provided: only extra complication, with resulting risk of confusion by all the opposites. The simplicity of Fig. A is all that is needed, with a minimum of such risk. The analogy between inductive and capacitive reactance is brought out, instead of an artificial difference between them being created. And the diagram represents both lots of halfcycles.

The same phasor diagram would still hold good if the top box contained a pure inductance and the bottom one a pure capacitance; but which one would Mr Gray then decide to call the load, and why?

In these days of increasing difficulty in packing a rapidly expanding electrical engineering curriculum into a given number of

years, anything that would shorten and clarify the early stages would surely be worth while. It is easy to show that there is considerable scope for doing this. Opposite *V* and *E*, and arrowheads with all their resulting diversity of practice and significance, are just two features that can thankfully be dispensed with, as they are in the integrated plan outlined in the article referred to.—Yours faithfully,

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*It is inept to denote it by i_{ab} because in general there may be a number of different currents flowing from *a* to *b*. The duals of the circuit junctions *a* and *b* are the circuit meshes each side of the current, say *C* and *D*, as Bow realised in mechanical analogy nearly a century ago. Publication of the corresponding electrical notation appears to have had to wait until 1954—see *Electrical Review*, 1st Jan. 1954, p. 14